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Thermodynamic aspects of rock friction

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ÖSSZEFOGLALÁS: Összefoglaljuk a kövek súrlódására vonatkozó kísérletei tapasztalatokat, különös tekintettel a földrengések kialakulásánál fontos lassú, állandó sebességű relatív mozgást végző felületekre. Ezután a kísérleti tapasztalatokat az empirikus modellek fényében termodinamikai szempontból elemezzük és megadunk egy egyszerű leírást, amely kompatibilis a második főtétellel.

Kulcsszavak: sebesség és állapotfüggő surlódás, nemegyensúlyi termodinamika

ABSTRACT: The summary of sliding rock friction experiments is given, with the emphasis on steady slow relative motion of the surfaces, that is important from the point of view of earthquakes. These observations are analyzed from a thermodynamic point of view and a simple model is developed that is compatible with the requirements of the second law.

keywords: rate and state dependent friction, non-equilibrium thermodynamics

1 INTRODUCTION

Frictional force arises between two contacting objects and their relative motion depends on it. Thus friction has been an important subject in several areas of mechanics, e.g. in mechanical engineering. In geophysics, relative motion at plate boundary is considered as frictional sliding. The interaction of friction and the rock material of plates leads to earthquakes. Large earthquakes generate propagating coseismic deformation as seismic waves and they can cause disasters, therefore it would be useful to know when, where, and how large earthquakes will occur in order to mitigate the damages by approapriate measures. This is a difficult problem.

The difficulty partially comes from the incomplete understanding of earthquake mechanism. Earthquakes are caused by fracture and deformation in the earth's crust, therefore it is necessary to understand the deformation before earthquakes and how it is fractured during the earthquake. One may think that the difficulty of earthquake mechanism comes from the instability of the fracture. However, the incompleteness of the deformation theory of the medium with external force is also essential. For example, crust consists of crystallized rock, grains with various size (e.g., sand, mud), consequently, the theory should explain the deformation by the applied external stress based on a constitutive law (with temperature if possible). However, present theory is based mainly on elasticity and the constitutive laws are empirical. We mentioned friction and earthquake in the above two paragraphs. Their relation can be understood on the example of metal friction. In tribology, one cannot avoid erosion of frictional surfaces even if they are polished smoothly. The wear off the surface, the erosion, is related to the accumulation and the release of stresses, which leads to earthquakes. The instability of the friction laws is the first step to understand the mechanism of earthquake.

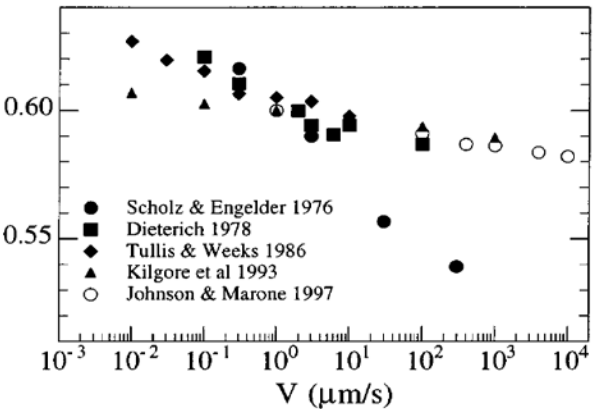
Properties of rock friction have been investigated mainly by laboratory experiments, and empirical equations have been proposed [e.g., Dieterich (1979), Ruina (1983), Nagata et al., (2012)]. Some researchers proposed models to explain the mechanism assuming a special shape of the frictional contact surface [Brechet and Estrin (1994)]. The real surfaces, however, do not have any special shape but have small particles of the medium between them. Moreover, the contacted rocks can be eroded during the frictional sliding as we mentioned above. Thus their assumption is insufficient and another theory is needed. Accordingly the understanding of the universal aspects of the mechanism of friction requires a thermodynamic framework. Thermodynamics is related to stability (Matolcsi, 2005), therefore frictional instabilities are challenging when the aim is to understand the role of second law in case of dissipation.

In this paper, we focus the change in dynamic friction depending on the shear loading rate [Dieterich (1979), Ruina (1983), Nagata et al., (2012)] as a first step of constructing theory that is compatible with the stability of thermodynamics. First, we summarize the qualitative properties in experimental data of rock friction. Second, we mention the key points of thermodyamical modeling and equations of the model in Section 3. After that we discuss the results in Section 4.

2 SUMMARY OF ROCK EXPERIMENTS AND EMPIRICAL CONSTITUTIVE LAWS

The rock sliding friction experiments are simulated by the so called rate- and state-dependent friction laws. These unify the results obtained from two types of rock experiments; the first one is the time dependence of static coefficient of friction [Dieterich (1972)], and the second one is sliding velocity dependence of the dynamic coefficient of friction [Dieterich (1978)].

The properties of dynamic friction are as follows; 1) frictional coefficient in case of stable sliding conditions with a constant loading rate depends on the logarithm of the loading rate (Figure 1a); 2) the magnitude of the instantaneous jump of the frictional coefficient depends on the change of the loading rate; 3) the following relaxation of frictional coefficients to a stable stationary value is also dependent on the instantaneous change of loading rate (Figure 1b); 4) oscillation occurs in some cases (e.g., large loading rate, polished surfaces, thin sand interface layer between the samples) [Marone et al. (1990)] (Figure 1c).



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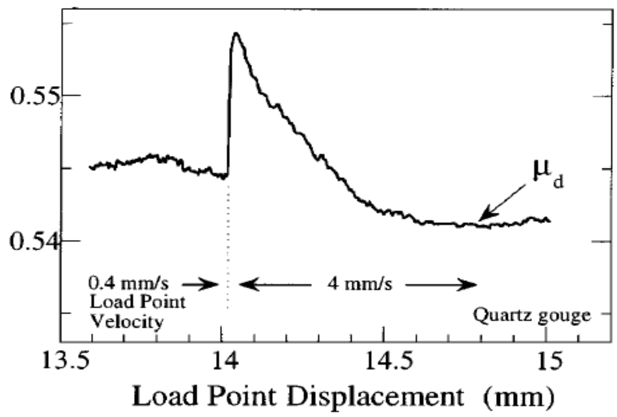

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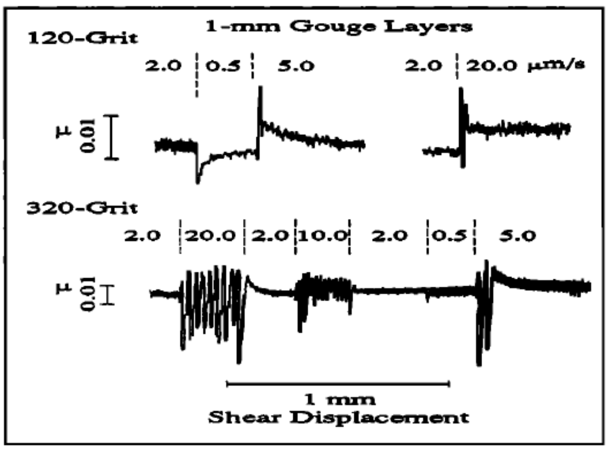

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Figure 1. (a) Dynamic coefficient of friction versus slip velocity for initially bare rock surfaces (solid symbols) and granular fault gouge (open symbols). The data have been offset to μ=0.6 at 1μm/s (Marone, 1998). (b) Dynamic coefficient of friction changing with loading velocity jump for a 3-mm-thick layer of quartz gouge sheared under nominally dry conditions at 25-Mpa normal stress (Marone, 1998) (c) The coefficients of friction versus displacement for 1-mm thick layers sheared within 120 and 320 grid surfaces. Load point velocity (μm/s) is given above each plots (Marone et al., 1990*). (A dinamikus súrlódási együttható a csúszási sebesség függvényében, kezdetben sima felületek (kitöltött jelek) illetve szemcsés felület esetén (üres jelek). Az adatok μ=0.6- ra kiegyenlítettek, 1μm/s csúszási sebességnél. (b) A dinamikus súrlódási együttható változása sebességugrás esetén, egy 3mm vastagságú kvarc rés névlegesen száraz körülmények közötti csúsztatása esetén 25 MPa merőleges nyomással (Marone 1998) (c) A súrlódási együtthatók az elmozdulás függvényében 1mm vastag rétegek csúsztatása esetén. A sebességek (μm/s) az egyes görbék fölött olvashatóak (Marone et al. (1990))*

The properties can be reproduced by using two equations except the oscillation. The first one is the constitutive law (1), expressing the relation between frictional coefficient μ and sliding velocity V with an additional variable called state variable θ. The second one is the evolution law (2) expressing the time evolution of state variable depending on the sliding velocity. The followings are by Dieterich (1979)

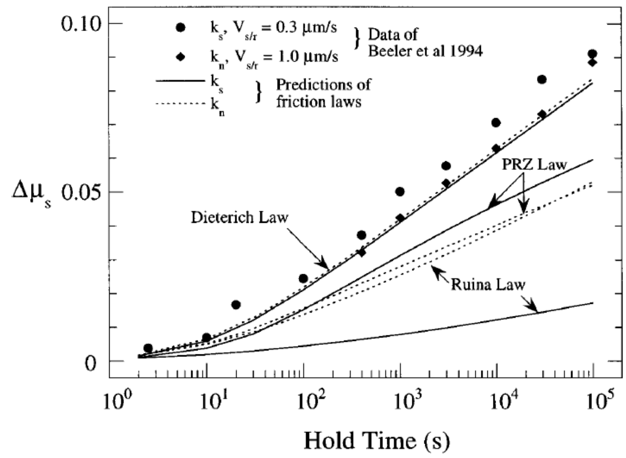
$$\tau = \tau_0 + a\sigma \ln\left(\frac{V}{V_0}\right) + b\sigma \ln\left(\frac{V_0\theta}{D_c}\right), \tag{1}$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c} \tag{2}$$

where τ is a shear stress, σ is a normal stress, μ_0 is a constant for steady-state slip at reference sliding velocity V_0, D_c is a critical slip distance, and t is time. The other evolution law proposed by Ruina (1983) is

$$\frac{d\theta}{dt} = -\frac{V\theta}{D_c}\ln\left(\frac{V\theta}{D_c}\right). \tag{3}$$

The static friction is better reproduced by the equations of Dieterich (1979) (Figure 2a), and the dynamic one is by those of Ruina (1983) (Figure 2b). Thus another versions have been proposed (e.g., Perrin et al. (1995), Kato and Tullis (2001)). However, none of them can reproduce the experimental data completely. Therefore these equations do not express a proper mechanism of friction.



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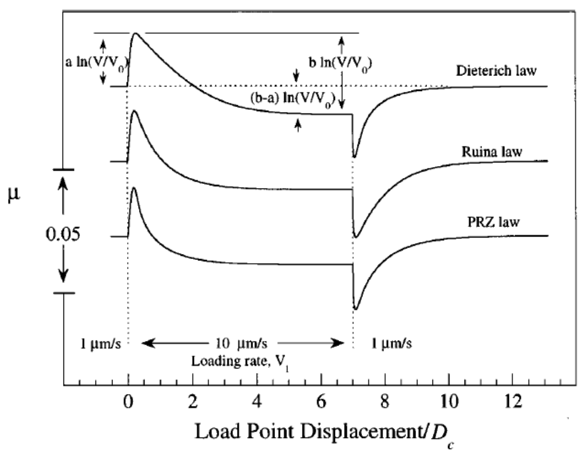

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Figure 2. (a) Data (solid symbols) and the predictions (lines) about time dependency of the change in static friction for initially bare granite surfaces by using data in Beeler et al. (1994). The stiffnesses, normalized by the normal stress are k_n=0.002μm^{-1} and k_s=0.072μm^{-1}, a=0.008, b=0.009, and D_c=3.0μm (Marone, 1998) (b) Simulated coefficients of friction versus normalized displacement by D_c using three evolution laws, respectively (see Perrin et al. (1995) about the details of PRZ law). They are calculated with the following parameters; V_0 = 1μm/s, a=0.01, b=0.015, D_c=20μm, and k=0.01μm^{-1} (Marone, 1998) *((a) Adatok (kitöltött szimbólumok) és modell illesztés (vonalak) a statikus súrlódási együttható időfüggő változására, eredetileg sima gránit felületek esetén Beeler et al (1994) adataival. A merőleges feszültségre normált merevségi együtthatók k_n=0.002μm^{-1} and* k_s=0.072μm^{-1}, a=0.008, *b=0.009, and* D_c=3.0μ*m (Marone, 1998) (b) Modellezett csúszási együtthatók a* D_c*–vel normalizált elmozdulás függvényében három fejlődési egyenletet használva (a PRZ törvény részleteit illetően lásd Perrin et al. (1995)). A paraméterek a következőek:* V_0 = 1μm/s, a=0.01, b=0.015, D_c=20μm, and k=0.01μ*m*$^{-1}$ *(Marone, 1998))*

There are two particular problematic aspects in these laws; the first one is the real meaning of state variable, and the second one is that sliding velocity at the frictional surface is assumed to equal to the loading velocity of the contacted rock samples in this equations.

Thus these are improved in a recent version [Nagata et al., (2012)]:

$$\tau = \Theta + a\sigma \ln\left(\frac{V}{V_0}\right) \tag{4}$$

$$\frac{d\Theta}{dt} = \frac{b\sigma}{D_c/V_0}\exp\left(-\frac{\Theta-\Theta_0}{b\sigma}\right) - \frac{b\sigma}{D_c}V - c\frac{d\tau}{dt} \tag{5}$$

In these equations, state variable θ is replaced to shear strength Θ with a reformulation of Nakatani (2001), and sliding velocity V is different from loading velocity V_l and it is estimated by another way with shear stress τ and system stiffness k. The third term in the right side of eq.(5) is a stress weakening term, a proposition based on the estimation of shear strength by the acoustic wave transmissivity experiment [Nagata et al., (2012)]. These equations reproduce also oscillation of the shear stress and fit the data better than the previous versions (Figure 3)

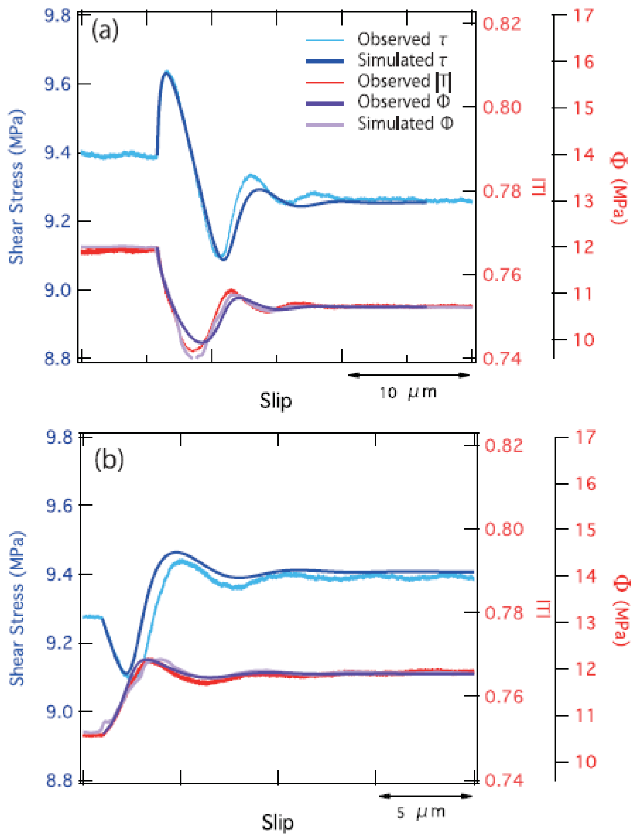

Figure 3. Observed data of shear stress τ and shear strength Θ estimated by using eq.(4) with data of τ and V, and their simulated ones during velocity step test (a) from 0.01μm/s to 0.1μm/s and (b) from 0.1μm/s to 0.01μm/s, respectively. Observed acoustic transmissivity |T| is also shown. The simulation are done by using eq. (5) and a=0.05, b=0.056, c=2, D_c=0.33μm, k=0.2Mpa/μm, σ=10Mpa, and V_0=1μm/s (Nagata et al., 2012). *(A τ nyírófeszültség és az Θ nyírószilárdság a (4) egyenlet alapján τ és V adatok, illetve szimulált értékeik felhasználásával, sebességugrásos kísérletek esetén (a) 0.01μm/s -ről 0.1μm/s-ra, (b) 0.1μm/s-ról 0.01μm/s-ra. A megfigyel akusztikus tranzitivitást is mutatja az ábra. A szimulációk az (5) egyenlet alapján* a=0.05, b=0.056, c=2, D_c=0.33μm, k=0.2Mpa/μm, σ=10Mpa *és* V_0=1μm/s *paraméterekkel történtek (Nagata et al., 2012).)*

These equations are useful for reproducing the experimental result easily, but their origin is empirical, and their form, especially logarithmic dependency, is not explained. Nakatani (2001) proposed thermal activation as the origin, and it is assumed also in a theoretical model of metal friction [Brechet and Estrin (1994)]. The origin of thermal activation theory is chemical [Eyring (1936)], but in our case the microscopic background is different. The basic components in rock friction are crystals of minerals and their fragments, and then geometric consideration of their mass deformation with applied force is also important. It is reasonable considering also from the fact that the rock sample was prepared to level out of the contacted surface by sliding, after the grinding before the experiments by Nagata et al. (2012) (Nagata, personal communication).

On the other hand, the deformation of rock fragments is investigated by experiments with simulated gouge (that is, sand particles) between the rock samples, and the gouge becomes to be fine-grained and compacted in vertical direction during the shear displacement [Mair and Marone (1999)] (Figure 4). The rate of the compaction depends on the logarithm of time just after the change of loading velocity in velocity-step test, and the compaction with the displacement depends on the loading velocity negatively (Figure 5). The compaction dependency on the loading velocity can be seen in the experiments with bare (that is, without simulated gouge) surface of the rock samples [Beeler and Tullis (1997)] (Figure 6). They show opposite dependency on loading velocity; shear stress shows velocity strengthening (that is, increase) with velocity increase in the case with simulated gouge, and it shows velocity weakening in the case of bare surface. However their change of compaction with displacement seems to be similar, and only the absolute values are different.

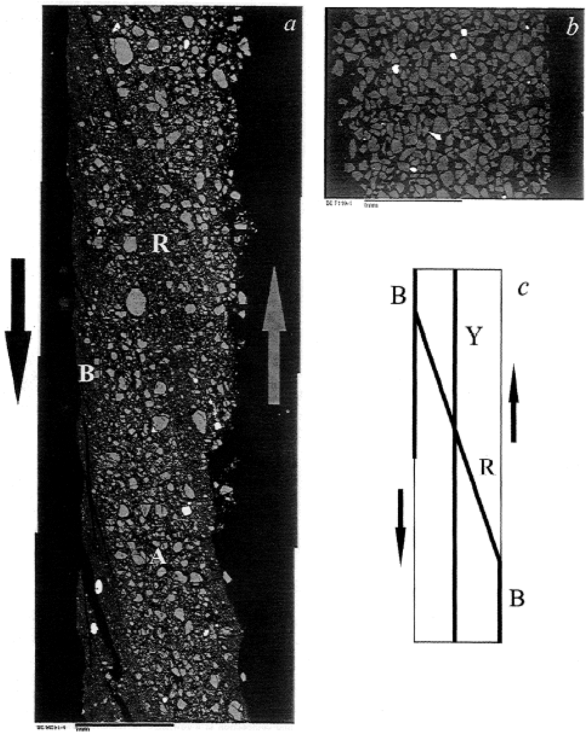

Figure 4. (a) Backscattered scanning electron micrograph (SEM) of a quartz gouge layer (initially 3mm wide) deformed at 50Mpa, 0.5mm/s, shear displacement of 20mm. The arrows shows the direction of shear. Region A indicates relics of relatively undeformed material, B shows boundary shear, and R shows oblique (Riedel) shear. (b) SEM of an undeformed quartz gouge layer. (c) Schematic diagram of some typical shear localization in a gouge zone. The shear displacement progresses from R (Riedel shears), to B (boundary shears), and finally to Y (boundary-parallel shears) (Mair and Marone, 1999). *((a) Visszaszórásos pásztázó electronmikroszkópos felvétel (SEM) a kezdetben 3 mm vastag súrlódó kvarcrétegről, amely 50Mpa normálnyomáson és 0.5mm/s sebességgel, 20mm teljes nyírási hosszon deformálódott. A nyilak a nyírási irányt mutatják. Az A terület viszonylag deformálatlan anyagmaradványokat, a B felületi nyírást, a C pedig ferde (Riedel) nyírási sávokat jelez. (b) Hasonló felvétel deformálatlan kvarrétegről. (c) A súrlódó zónában látható tipikus nyírási lokalizációk sematikus ábrája. A nyírási elmozdulás először a Riedel zónákban (R) jelentkezik, majd felületi nyírásba (B) és végül felülettel párhuzamos nyírásba (Y) megy át.)*

3 THERMODYNAMICAL MODELING OF FRICTIONAL LAYER

We need to mention two key points of view before we model the deformation of frictional layer thermodynamically. The first one is how we think in the thermodynamic way, and the second one is how we consider the relation between deformation of rock fragments and the applied external force by the rock samples. We distinguish in between two modeling levels: the continuum and the discrete. At the continuum level the friction layer, and the surrounding rock mass is modeled as a continuum. At the discrete level we consider the rock sample as a single entity. The first level is a more detailed one and we have seen in the previous section, that several details and mechanisms are revealed only in this level. On the other hand the second case, the level of rate- and state friction laws, is our usual way of modeling friction. The two levels are interdependent, now we use continuum aspects to motivate a discrete thermodynamic approach.

3.1 *Continuum aspects – mechanisms of dissipation*

There are several aspects of a thermodynamical approach, and we show them with the correspondence to friction as follows.

The first one is stability. Thermodynamic phenomena can be categorized as equilibrium, steady-state, and non-equilibrium from the point of view of time dependency. The aforementioned properties of dynamic friction are regarded as follows; 1) stable sliding is steady-state, and 2)-4) shear stress reaction in velocity-step test and oscillation is non-equilibrium.

Second one is dissipation. It is directly related to the difference between equilibrium and non-equilibrium. Here we have to consider carefully the different occurrences of dissipation. Dissipation occurs when energy flows out accompanying the mass or by heat conduction to rock samples. Energy flows can occur by mass decrease or by only energy decrease with the particle rolling out from the frictional layer although the mass itself does not change because of the mass flowing into the layer without any mechanical energy. The latter one leads to that frictional displacement is proportional to the rotational energy dissipated from the system and it is irreversible. It can be important for modeling, however, it may be neglected in the case of steady state and we can consider only energy balance of the system.

The third one is entropy. In mechanical dominated cases, e.g. without heat conduction, entropy change is energy change/temperature, and it is regarded as (irreversible energy change)/(the ratio of irreversible energy to reversible one). In frictional case, it can be defined as (rotational energy change)/(the ratio of rotational energy to compaction energy like a volumetric elastic energy) if energy change by heat with temperature can be neglected.

3.2 *Discrete aspects – thermodynamics of rate- and state.*

The considerations in the previous subsection show, that loss of elastic stability and the development of shear bands are fundamental in rock friction. The corresponding continuum description should introduce the loss of stability, a kind of plasticity, in case of an essentially granular media. Moreover, rheological effects, creep and relaxation, cannot be neglected. Our thermodynamic treatment is discrete. The whole continuum is modeled as a single thermomechanical body and the difficulties are hidden in the properties of the contact mechanism. However, our contact model is based on a simplification of a particular thermodynamic compatible plasticity theory where creep and rheology can be included in a natural way.

In the following we develop a treatment of sliding friction in the framework of non-equilibrium thermodynamics. The particular approach is motivated by the thermodynamics of plasticity theories. Related works with conceptually similar researches are Houlsby and Puzrin (2006) which considers the ideas of Ziegler (1981) as a starting point. These ideas were partially developed in Ván (2010), where the connection to plasticity is introduced and the yield criteria and flow rules were interpreted with the help of simple nonlinear Onsager conductivity relations. In the following we add one particular ingredient, quantities plastic strain rate, that is the key to distinguish the permanent and elastic parts of the strain in plasticity. In case of friction this leads to the distinction of permanent displacement of the surfaces and an elastic, recoverable counterpart.

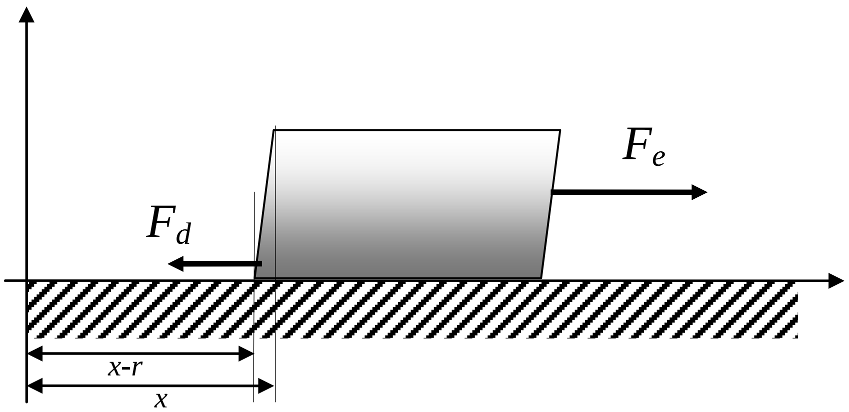

Figure 5 Sliding thermomechanical body *(Csúszó termomechanikai test)*

Let us consider a body on a horizontal surface with mass *m*. There are two forces that determine the motion of the body: the external force F_e, and the *damping force* F_d, due to friction (Figure 5). The position of the body is characterized and it is denoted by x. The body is not considered completely rigid, but we assume that one particular material point of the body may characterize its instantaneous position. The equation of motion is

$$m\ddot{x} = F_e - F_d. \tag{6}$$

Moreover we assume that the energy of the body, E, is conserved without the external force. Therefore

$$\dot{E} = F_e \dot{x}. \tag{7}$$

We will see, that then thermodynamics requires that the damping force contributes only to the internal energy of the body. Our body is energetically an open system, however, we do not calculate directly the energy balance of the environment. We assume that the external force accelerates the body and also that the body is deformable. Therefore we distinguish two energies of the body, the kinetic and the elastic energies.

The internal energy of the body, U, is the difference of the total energy, the kinetic energy and the elastic energy:

$$U = E - m\frac{\dot{x}^2}{2} - k\frac{r^2}{2}. \tag{8}$$

Here k is the parameter of the elastic energy per the elastic deformation r and we assumed a particular kinematic condition: the instantaneous position of the body is the sum of a permanent and a recoverable elastic displacement. A convenient method of their distinction is an additive separation of the displacement rates.

$$v = \dot{x} = \dot{r} + z. \tag{9}$$

where v is the rate of the position x, z is the rate of a permanent displacement.

This kinematical condition is well accepted method to introduce the distinction of plastic and elastic strain rates in plasticity (see e.g. Fülöp and Ván (2012), Rusinko and Rusinko (2011)). The entropy of the body, S, is the function of the internal energy and the derivative of entropy by the internal energy is the reciprocal temperature $1/T$. Therefore the calculation of the entropy rate is simple and the dissipation, the product of the entropy rate and the temperature, follows:

$$\dot{S}(U) = \frac{1}{T}\left(F_e\dot{x} - mv\dot{v} - kr\dot{r}\right) \geq 0 \quad \Rightarrow \quad T\dot{S} = F_d v - kr\dot{r} \geq 0 \tag{10}$$

Here the damping force and also the elastic strain rate are the constitutive quantities to be determined in accordance with the requirement of nonnegative entropy production. The simplest solution of the inequality assumes linear relationships between the thermodynamic fluxes and forces with coefficients l_1, l_2, l_{12},and l_{21}. Therefore

$$\begin{aligned} F_d &= l_1\dot{x} - l_{12}kr, \\ \dot{r} &= l_{21}\dot{x} - l_2 kr. \end{aligned} \tag{11}$$

Here the symmetric part of the matrix of the constant thermodynamic coefficients is positive definite, therefore $l_1>0$, $l_2>0$, $l_1\, l_2 - l^2 \geq 0$, where $l= (l_{12} + l_{21})/2$. Substituting the Newton law, eq. (6), to eq. (11) we obtain:

$$\begin{aligned} m\ddot{x} &= F_e - l_1\dot{x} + l_{12}kr, \\ \dot{r} &= l_{21}\dot{x} - l_2 kr. \end{aligned} \tag{12}$$

One may eliminate the r elastic strain and obtain the following third order differential equation:

$$\dot{F}_e + l_2 k F_e = m\dddot{x} + (l_1 + l_2 km)\ddot{x} - (l_1 l_2 + l^2 - a^2)k\dot{x}, \tag{13}$$

or, equivalently also the x displacement and keep the elastic strain:

$$\frac{m}{l-a}\ddot{r} = F_e - \frac{1}{l-a}(l_1 + l_2 km)\dot{r} - \frac{1}{l-a}(l_1 l_2 - l^2 + a^2)kr, \tag{14}$$

where $a = (l_{12} - l_{21})/2$. In these equations the l_1 coefficient characterizes direct mechanical damping effects. If it is constant, then it can be identified the classical damping which is proportional in magnitude and opposite in direction to the velocity. We can see, that the structure of the equations is identical with that of the Verhás body (inertial Poynting-Thomson) that proved to be fundamental in thermodynamic rheology (see e.g. Fülöp (2008) and Szarka et al (2008)).

However, in these equations Coulomb type friction does not appear. Therefore we introduce also a velocity dependent coefficient, as follows:

$$l_1(v) = \frac{\beta}{1 + \frac{\beta}{\alpha}|v|}. \tag{15}$$

Then the corresponding damping, that is $l_1 v$, tends to constant in case of high velocities and proportional to the velocity, in case of low velocities. This term is switching between the two characteristic behaviors, the transition point is determined by parameters, α and β.

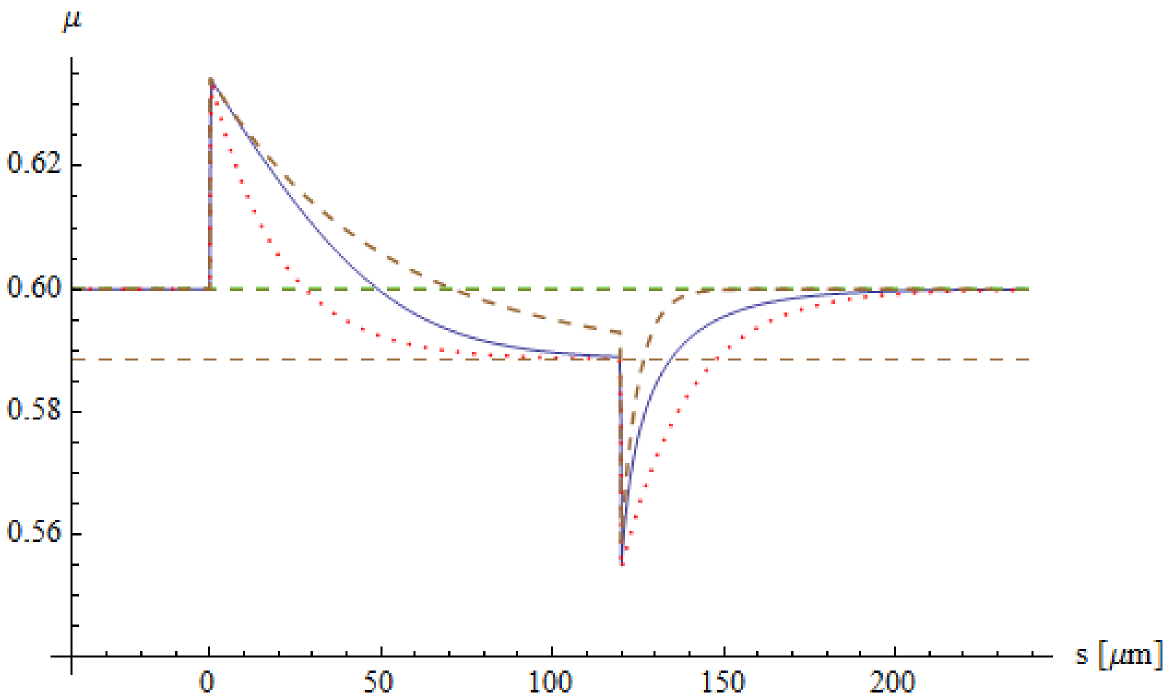

Figure 6. Comparison of the rate- and state friction laws for velocity jumps and steady-state velocity weakening experiments. The Dieterich model (solid line), Ruina model (dotted line) and the thermodynamic one (dashed line) show different relaxations. *(A sebesség- és állapotfüggő surlódási törvények összehasonlítása sebességugrásos és sebességgyengülést mutató esetben. A Dieterich modell (folytonos vonal), Ruina modell (pontozott) és a termodinamikai modell (szaggatott) különböző relaxációt mutatnak.)*

Finally on Figure 6 we compare the performance of the classical Dieterich (eqs. (1)-(2)) and Ruina (eqs. (1) and (3)) models with our thermodynamic suggestion. The parameters of the classical models are from Marone (1998) (see Figure 2b), where it was used for fitting experimental data. The experimental data are not shown here. Some parameters and initial values of the discrete thermodynamical model (12)-(15) can be calculated from the stationary values of friction: $l_2 = 195.43 kg/s^2$, $\alpha = 0.644 \mu N$, $\beta = 10.09 kg/s$. Other parameters are given to show a comparable figure.

The most important difference between the models is the type of relaxation. The thermodynamic model shows and exponential one, but the classical models are based on a logarithmic relaxation, a different one in case of the two models.

4 DISCUSSION

Our simple thermodynamic model incorporated several aspects of rate- and state dependent friction into a uniform theoretical framework. The separation of dissipative and nondissipative parts, the meaning of the different parameters was clear. The most important advantage of our model is that the jump condition was a consequence of the evolution equation and was not a separate assumption. However, the number of the parameters was seemingly higher, and the logarithmic relaxation was not incorporated in our model. Introducing logarithmic terms in the theoretical framework as separate assumptions would be simple. However, the most important question for us to find a mechanism, an explanation of the origin of these terms. We have argued above that a more detailed analysis of the continuum aspects may lead to such an explanation.

ACKNOWLEGEMENTS

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